art and science at play

ZOMETOOL®

Platonic Solids

art and science at play

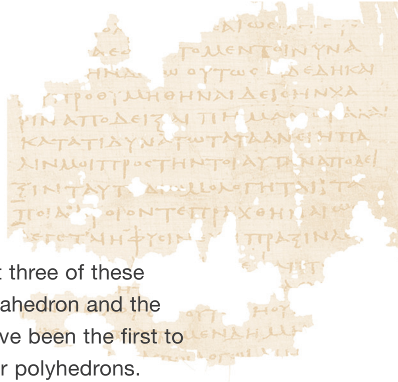
ZOMETOOL®

“Where there is matter, there is geometry”
(Johannes Kepler, Harmonices Mundi, 1619)

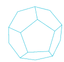
The five forms which we now refer to as the Platonic Solids are to be discovered not only in mathematics, art and architecture, but also in the world around us: within nature they are to be found in a wealth of crystalline structures as well as in living organisms, such as protozoa and viruses.

These forms have been known to mankind for thousands of years. The very earliest representations we know of - a series of intricately fashioned marble-like objects termed the Ashmolean carvings - are believed to be over 5,000 years old. Although these carvings do not correspond with the Platonic Solids in all details, they provide us with an inspiring insight into man's age-old fascination with the regular solids. The earliest pyramids were constructed at around the same time and bear evidence of the Ancient Egyptians’ knowledge of the tetrahedron, the hexahedron (cube) and the octahedron.

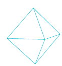
left and above: Radiolarians (protozoa), illustrations from Ernst Haeckel, 1879.

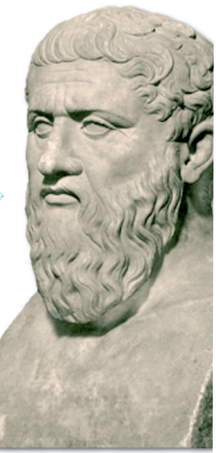


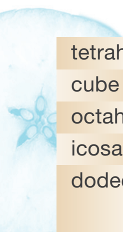
The Pythagoreans (ca. 550 BC) studied at least three of these solids: the tetrahedron, hexahedron and dodecahedron and the Athenian Theaitetos (ca. 415 - 369 BC) may have been the first to prove that there can be only five convex regular polyhedrons.



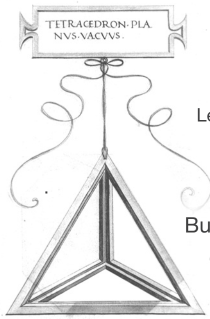
The Platonic Solids are named after Plato (ca. 423 - 347), the well-known Greek philosopher, mathematician and teacher whose writings have so greatly influenced Western thought, probably since he was one of the first to describe them together and use them as the building blocks of one of the earliest known ‘Theories of Everything’. In his system, the properties of the four classical elements were represented by the regular solids - and of the fifth solid (the dodecahedron) he stated that it was “used for embroidering the constellations on the whole heaven”. Later, Aristotle would associate it with the ‘quintessential’ element, aether.



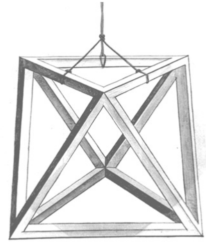




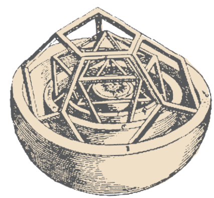
tetrahedron	fire	plasma
cube (hexahedron)	earth	solid
octahedron	air	gaseous
icosahedron	water	liquid
dodecahedron	aether	a reference to the cosmos



Leonardo da Vinci depicted all five Platonic Solids in "**Divina Proportione**".

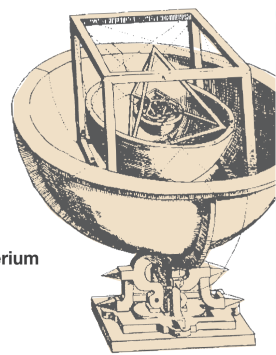


Building on this knowledge, Euclid (ca. 300 BC) described the Platonic Solids in the last book of his seminal work "Elements" and proved again the existence of exactly five regular, 'perfect' solids. He remarked that "No other figure, besides the said five figures, can be constructed by equilateral and equiangular figures equal to one another."



A simple summary shows that for each solid:

- all edges are of the same length
- all faces are made of the same regular polygon
- the same number of faces meet at each vertex
- all face edges meet at the same angle (dihedral angle)



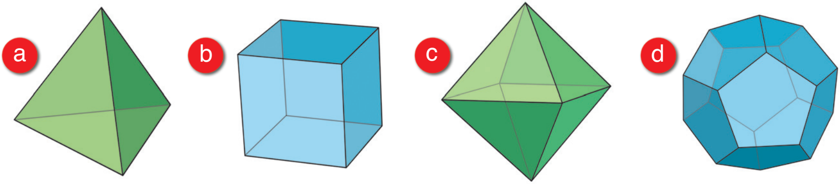
With the onset of the Renaissance, the unique properties of these forms ignited the curiosity of a wider group of intellectuals: Leonardo da Vinci (1452 - 1519) created a series of illustrations for Luca Pacioli's "Divina Proportione"

obove right: Kepler's illustrations of the Platonic Solids from the "**Mysterium Cosmographicum**", 1596 and right: his model of the solar system.

and the astronomer Johannes Kepler (1571 - 1630) based his concept of our solar system on the inherent relationships between the Platonic Solids and their corresponding spheres.

Kepler presented his theory of the planetary orbits in his revolutionary work “Mysterium Cosmographicum”. The Zometool kit “Kepler's Kosmos” offers an hommage to his cosmological theory, with a nested model of the five Platonic Solids proposed by the renowned mathematician John H. Conway.

The fascination with the Platonic Solids has continued to this day and in recent times, these intriguing forms have become increasingly integrated into our collective conciousness, with artists such as M.C. Escher, Salvador Dali and Olafur Eliasson incorporating them to great effect in their work.

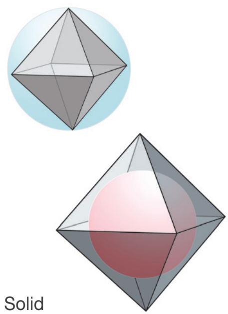


Defining characteristics of the five Platonic Solids

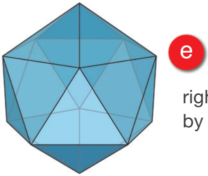
Platonic Solid	faces	2-D shape (polygon)	edges	vertices
a - tetrahedron	4	equilateral triangle	6	4
b - cube	6	square	12	8
c - octahedron	8	equilateral triangle	12	6
d - dodecahedron	12	regular pentagon	30	20
e - icosahedron	20	equilateral triangle	30	12

Each of the Platonic Solids shares a common centre point with 3 spheres:

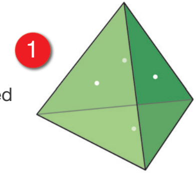
- **Circumsphere**. All vertices of a solid lie on the inner surface of a sphere. These corners represent the most symmetrical distribution for that number of points on the surface of their circumsphere.
- **Insphere**. This is contained by the solid and touches the centre of each of its faces.
- **Midsphere**. This passes through the midpoint of each of the edges of the solid.



right: circumsphere and insphere of an octahedron. To construct any Platonic Solid within a sphere, the vertices must simply be positioned at an equal distance from each other.

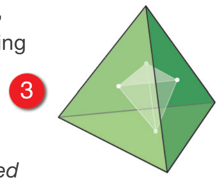


right: the dual solid of a tetrahedron can be constructed by connecting the adjacent midpoints of each face.



Duals

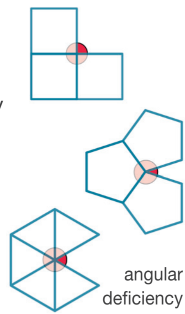
The Platonic Solids are also related to each other in several ways; one of the most important is the relationships of each solid with its dual. The vertices of each dual can be easily defined by placing a point in the center of each face of a polyhedron. By connecting each point to the next nearest point, the dual solid will materialise automatically. You will find a building guide for each solid and its dual in the ‘Step by Step’ section.



Why can there be only 5?

“*No other figure, besides the said five figures, can be constructed by equilateral and equiangular figures equal to one another.*”

One way to prove the accuracy of Euclid’s statement involves laying out polygons in a 2-D ‘net’ and using the angular deficiency between them to determine whether the net would remain flat or ‘pop’ into 3D. You can do this yourself using pencil and paper or Zometool components!



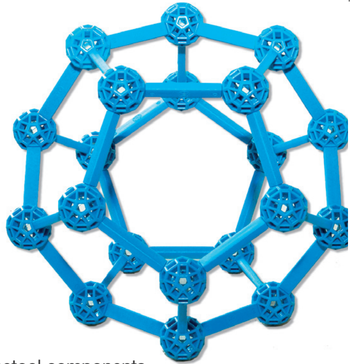
angular deficiency

To find out this and much more about the Platonic Solids visit:
zometool.com/pages/platonicsolids

art and science at play

ZOMETOOL glossary

Polyhedron: a solid in three dimensions with flat polygonal faces, straight edges and sharp vertices.
Dihedral angle: the angle created by two intersecting planes.
Angular deficiency: the failure of some angles to add up to the expected amount of 360° or 180°, when such angles in the plane would.
Pythagorean: member of a religious-philosophical sect which was founded in southern Italy in the early 6th. century BC.
Theatetus: ca. 415 - 369 BC. Greek mathematician who proved that there are only 5 regular convex solids. Plato, his friend, named a dialogue after him.
Plato: ca. 427 - 348 BC. Ancient Greek philosopher, student of Socrates and Aristoteles' teacher. Considered to be one of the most important figures in the history of Western Philosophy.
Euclid: Greek mathematician, lived in Alexandria ca. 3rd. century BC. Often refered to as the 'Father of Geometry', his 13-book treatise "Elements" is considered to be the most influential and successful textbook ever written.
Johannes Kepler: 1571 - 1630, German philosopher of nature, mathematician, astronomer and astrologer whose works include:
Harmonices Mundi: a work on astronomy.
Mysterium Cosmographicum: an early work, in which he attempted to prove a connection between the Platonic Solids and the solar system.
Leonardo da Vinci: 1452 - 1519, Italian painter, sculptor, architect, anatomist, mechanic, engineer and philosopher of nature. Considered to be the most famous universal scholar of all time.
M.C. Escher: 1898 - 1972, Dutch illustrator and graphic artist.
Salvador Dali: 1904 - 1998, Spanish painter, sculptor, stage designer and author.
Olaffur Eliasson: * 5. February 1967 in Copenhagen, Danish artist of Islandic descent, resident in Berlin and Copenhagen, whose work primarily revolves around natural physical phenomena (eg. light and water, movement and reflection).



To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
and eternity in an hour.

William Blake

Zometool Rules

1. If it works, it works perfectly…

… and if it doesn’t work, it doesn’t work at all. Don’t force Zometool components. You can bend a strut to fit it into a tight spot, but struts in finished models are always straight, never under tension.

2. Don’t break it apart, take it apart!

Take models apart by grasping a strut with your fingers and pushing the ball straight off with your thumb. Twisting balls or pulling models apart or crushing them can cause parts to break!

Whether you want to ask better questions or learn better answers, Zometool is your ticket to discovery and fun. From numeracy to nanotechnology, quasicrystals to quantum mechanics, the destination is always the same: understanding our amazing universe.

Notes on instructional schematics

All regular Platonic Solids can be built simultaneously - their duals one after another. White parts are used to build duals, black parts are used as ‘scaffolding’ - these may sometimes be removed.

Finished models can be constructed using standard-coloured parts, alternatively with coloured balls, as depicted.

Pyrite crystal

More under www.zometool.com