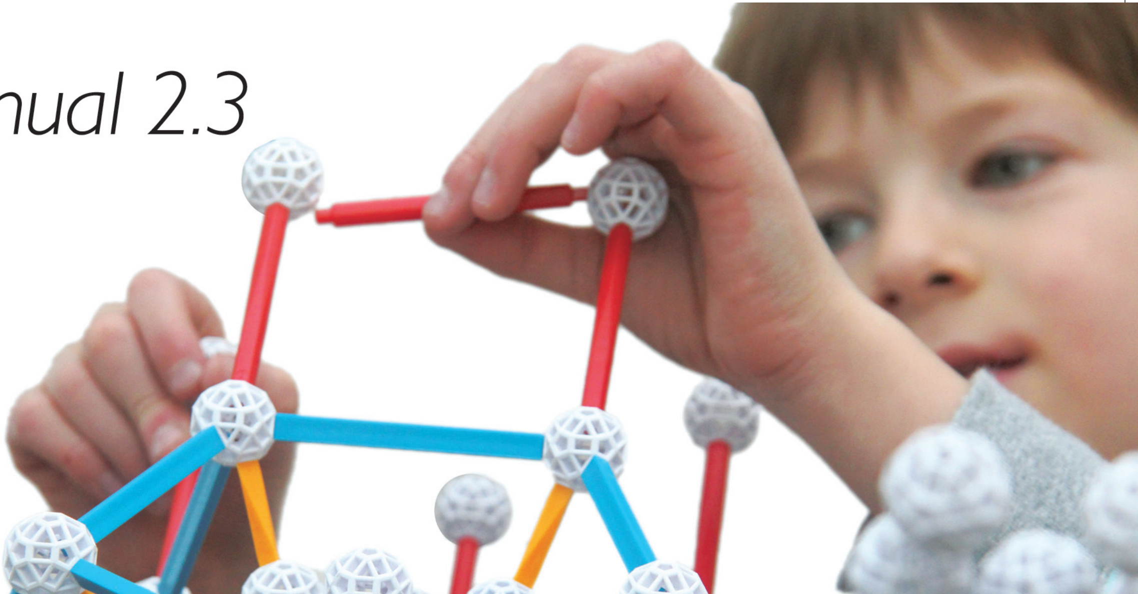
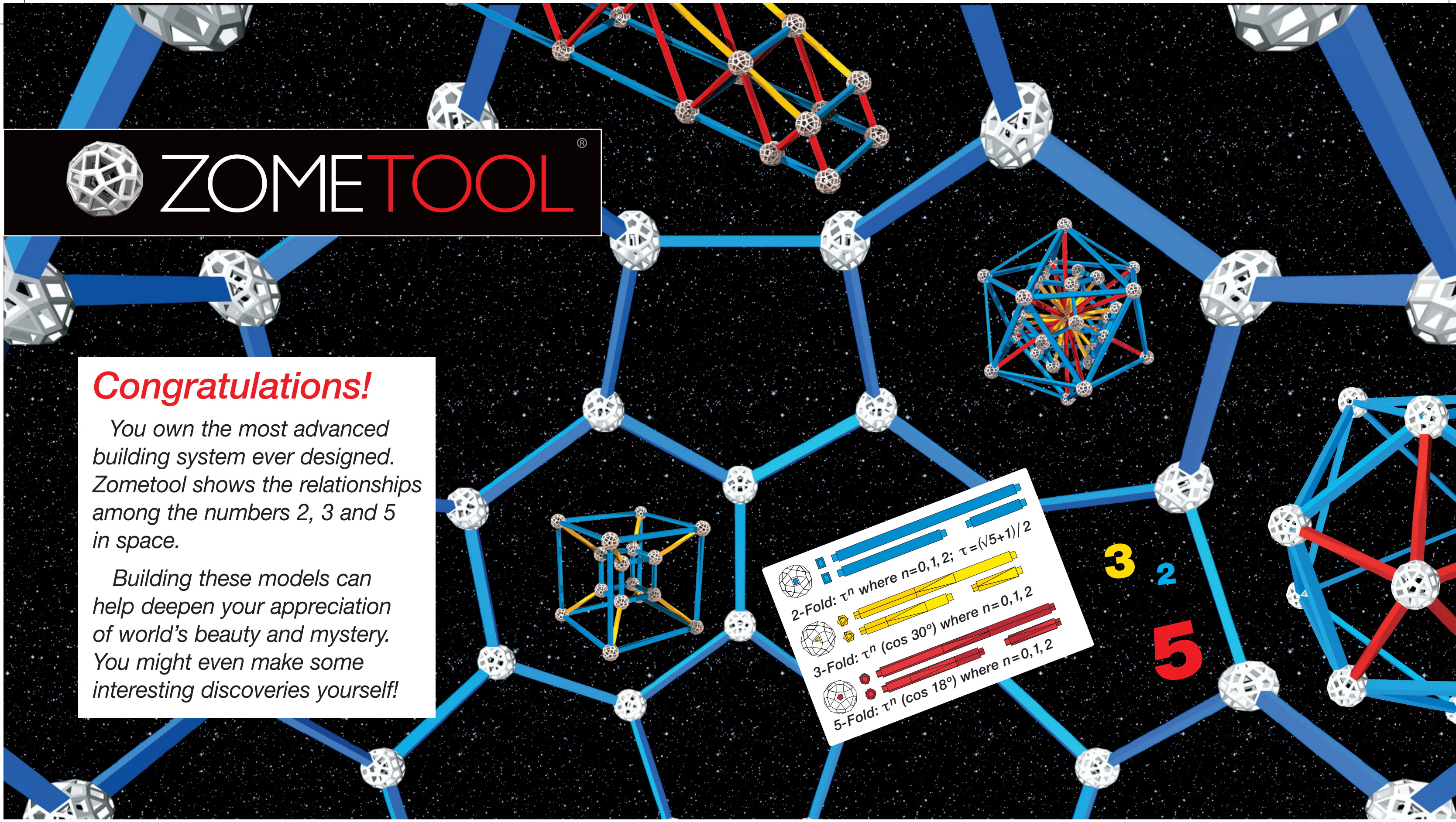
Manual 2.3



art and science at play

ZOMETOOL®

ZOMETOOL®



Congratulations!

You own the most advanced building system ever designed. Zometool shows the relationships among the numbers 2, 3 and 5 in space.

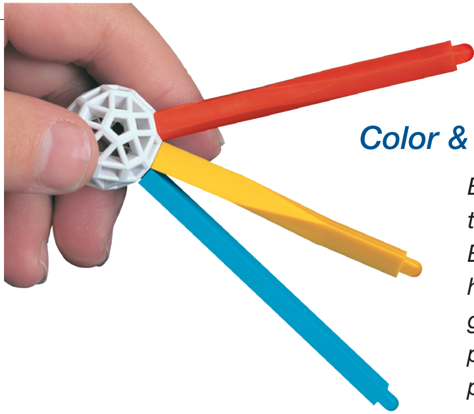
Building these models can help deepen your appreciation of world's beauty and mystery. You might even make some interesting discoveries yourself!

Zometool Manual 2.3 is an introduction to the amazing world of Zometool. In the following pages, you will discover that Zometool struts and balls build relationships in space that make beautiful models simple to build, and advanced concepts easier to understand.

Zome geometry is based on the underlying structure of nature. You'll find many references to the power of 2, 3 and 5. While this manual touches many mathematical concepts, it is not a textbook. Rather, we warmly invite you to further explore ideas presented here. The bibliography on the last page is a good place to start!

Kits can be expanded at any time. All models in this booklet can be built with any Zometool system kit.

Have fun!

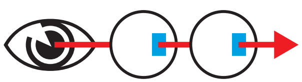


Color & Shape Show the Way!

Each Zometool strut connects to holes of matching shapes. Blue struts fit only rectangular holes, yellow struts only triangular holes, and red struts only pentagonal holes. This makes it possible to build even complex models with ease.

Zometool Rules!

If it works, it works perfectly.

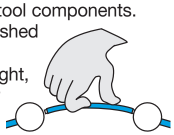


Which strut should I use?

You can tell which strut fits between two balls in a model by lining up the balls and looking through the holes: they show you the shape of the right strut.

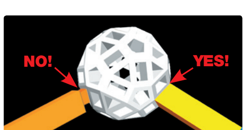
O.k. to bend struts?

You can bend a strut slightly to fit into a tight spot, but don't force Zometool components. Struts in finished models are always straight, never under tension.

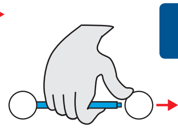


What about gaps?

Make sure each stub goes all the way into the hole. Tighten up your model as you go. Work locally, with one hand holding the ball and the other pushing the strut straight in.



Don't break it apart; take it apart!



How do I take it apart?

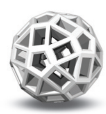
Take Zometool models apart by grasping a strut with your fingers and pushing the ball straight off with your thumb. Twisting balls, pulling models apart or crushing them can cause parts to break!

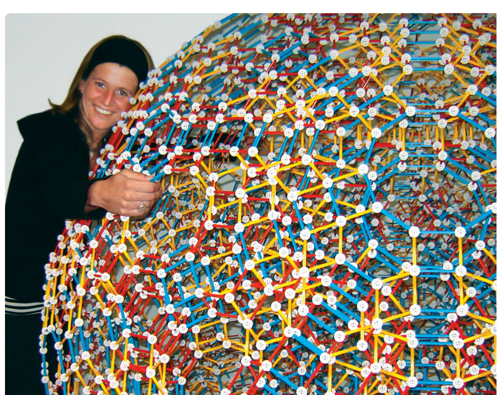
Don't crush models.

Building Tips

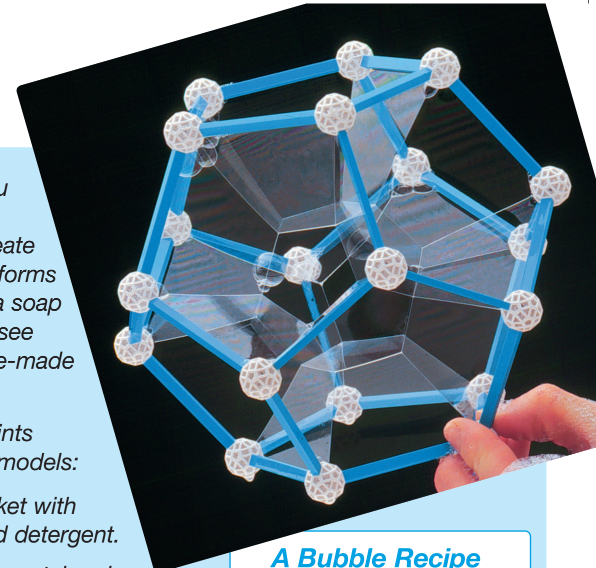
What if I break parts?

We replace accidentally broken parts for free: visit *www.zometool.com/broken-struts/* for details.





Bubbles!



Many models you can build with Zometool will create fantastic bubble forms when dipped in a soap bubble solution (see our favorite home-made formula below).

Here are a few hints for dipping your models:

- *Fill a deep bucket with water, <u>then</u> add detergent.*
- *Make sure the container is wide and deep enough for your largest model and your hand.*
- *Don't stir up the bubble solution more than necessary—no suds!*
- *Dip and lift your models slowly. Pop unwanted bubbles with a dry finger. Move bubbles around without popping by using a wet finger.*
- *Some models trap bubbles inside. The cube series below shows how different size bubbles can be trapped inside a model.*
- *Use a wet straw to add or remove bubbles.*
- *Have fun!*

A Bubble Recipe

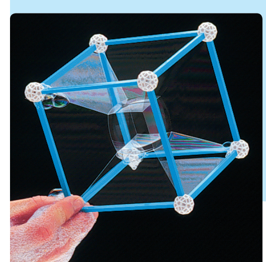
Start with 3 gallons warm water in an open container (like a 5-gallon bucket).

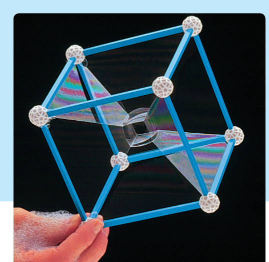
Carefully add $^{2}/_{3}$ cup *Dawn* or *Joy Ultra* dishwashing soap (to minimize foam).

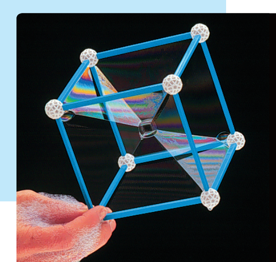
For tougher, longer-lasting bubbles, add 1 tablespoon glycerine *(available in any drugstore).*

Notes: *Add more soap if your bubbles are weak. For better results, allow the mixture to sit in an open container for up to one day before use.*

Thanks to Zometool user Kelly Nichols for bubble research.

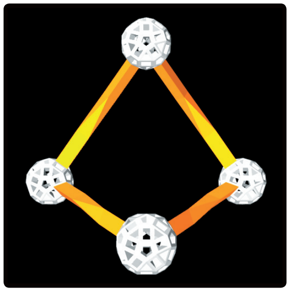






Amazing Bubbles!

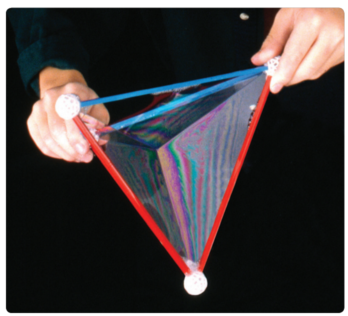
Each of these models can be easily built with almost any Zometool kit. And when they're dipped in bubble solution, they create beautiful bubble surfaces.



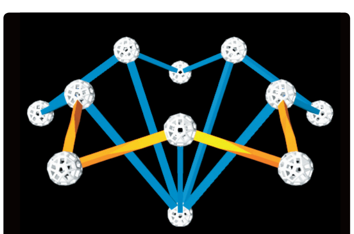
Saddle

*This model creates a bubble with a curved **(minimal)** surface. Can you find the highest low point of one curve meeting the lowest high point of another in this model? This is called a **saddle point.** Can you think of any buildings that use this shape?*

*Dip this 3-D triangle (tetra #34, page 8) in bubble solution and see a shadow of a 4-D triangle **(simplex)**. Just as the 3-D triangle is made of four 2-D triangles (count them!), the 4-D triangle is made of five 3-D triangles. Can you find them all?*



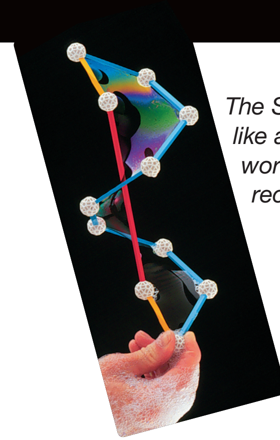
4-D Triangle



Flower

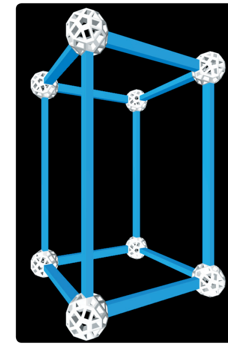
Dip this model to see five saddles joined. Why do many flowers have five petals? Can you think of other plants and animals with the number 5 in them? How about the numbers 3 & 2?

All of the models shown here can be built with nearly any Zometool kit.

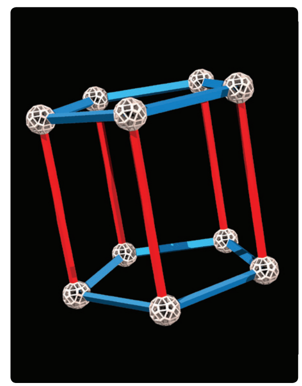


The Spiral bubble looks like a winding slide. Will it work if you take out the red strut?

For the "Cuboid," you must catch a bubble in the middle by dipping the model all the way, then only half way. Can you find all eight "squashed" 3-D rectangles that make the 4-D rectangle?

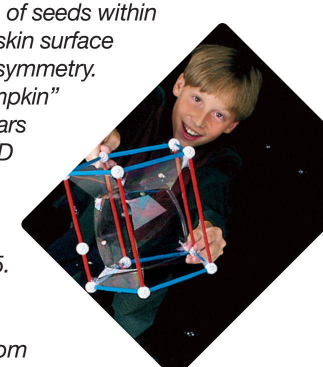


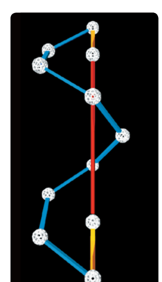
Cuboid



Pumpkin

A pumpkin encloses the maximum volume of seeds within the smallest skin surface using 5-fold symmetry. When a "pumpkin" bubble appears inside this 3-D pentagon, it is also constrained by the number 5.





Spiral

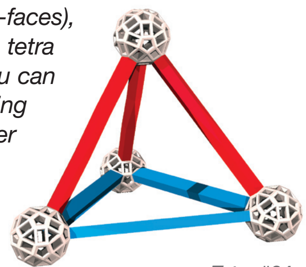
*Here is the Spiral model from the picture at the top of the page. Like the Flower model at left, it has **5-fold symmetry** because it has a pattern that repeats 5 times around its **axis** or center. Can you find the models that have **2-fold symmetry**? How about **3-fold symmetry**? Watch for these symmetric patterns as you build more complex models. It will make construction even easier!*

The Tetra Challenge

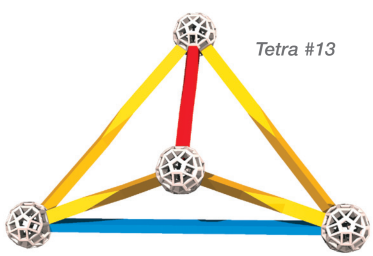
…Can You Build All 65?

*A 3-D triangle is called a **tetrahedron** (4-faces), or **tetra** for short. You have already seen tetra #34 in the Bubble Models on page 6. You can build 65 different **tetrahedra**, not including mirror images and flat ones. (We consider flat models to be 2-D shadows of 3-D triangles.) Tetras usually use 4 nodes and 6 struts, except for a few (like Tetra #55) with jointed edges.*

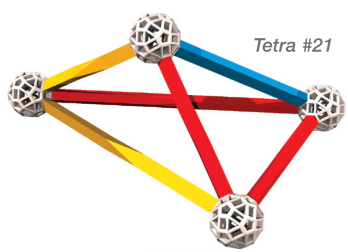
Try making bubbles with these models too!



Tetra #34

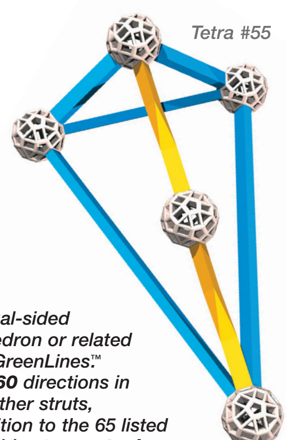


Tetra #13

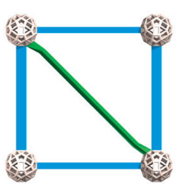


Tetra #21

*The tetrahedron and the **octahedron** (8 faces) are the basis for many strong structures. How many tetras are in the Pyramid on page 23? The Pyramid is part of an **oct-tet truss**. You can also build 65 octahedra in Zometool. So you can build 65 different oct-tet truss systems!*

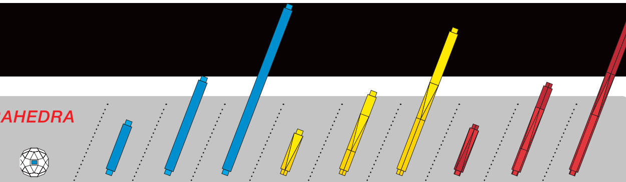


Tetra #55



***Note:** To build a **regular** or equal-sided tetrahedron (or a regular octahedron or related models,) you'll need Zometool GreenLines.™ GreenLines™ add an additional **60** directions in Zometool space and, with the other struts, can build 245 tetrahedra in addition to the 65 listed here. This advanced kit is available at **zometool.com**.*

65 Tetrahedra



Tetra #	Balls	B0	B1	B2	Y0	Y1	Y2	R0	R1	R2
1	4			1		1	1	1	1	1
2	5			2			2	1	1	1
3	4			1		1	1	1	2	
4	4			2			3	1		
5	4		1	1			3	1		
6	4		1	2			2	1		
7	4		1	2			1	2		
8	5		1	3			2	1		
9	4		2	1			1	2		
10	4			3			2	1		
11	4		1	1		3				1
12	4			1		2		1	2	
13	4			1		2	2		1	
14	4		2	1		3				
15	4			3		2				1
16	4		1	2		3				
17	4		1			1	1	1	1	1
18	4		2				1	3		
19	4		1		1	1	1	1	1	
20	4		1			2	1	1	1	
21	4		1			1	1		2	1
22	4		3				2	1		
23	4	1	2				1	2		
24	4		2	1			2	1		
25	4		1			1		2	2	
26	4		2		1				3	
27	4		1				2		2	1
28	4		1			2		1	2	
29	4		1			2	1		2	
30	4		1	2					3	
31	4	1	2		1				2	
32	4		2	1					3	
33	4	2	1		1				2	
34	4		3						3	
35	4		1			2	2		1	
36	4		2	1			3			
37	4		1	2			3			
38	4	1	2			2				1
39	4			1			2		2	1
40	4					1	2		2	1
41	4			1		1	2		1	1
42	4					2	1	1	2	
43	4						2		2	2
44	4			3					3	
45	4		2	1	1				2	
46	4			1		1	2		2	
47	4	2	1		2				1	
48	4	1	2		2				1	
49	4		2			3				1
50	4		3			2				1
51	5		3	2					2	
52	5		2	3					2	
53	5		2	3					2	
54	5		3	2					2	
55	5	2	1	2		2				
56	4		2			4				
57	4		3			3				
58	4	2	4							
59	4	1	4	1						
60	4	4	2							
61	4	3	3							
62	4	1	5							
63	4	3	3							
64	4		5	1						
65	7	1	1	1		2		2	2	

Building on Shape & Color

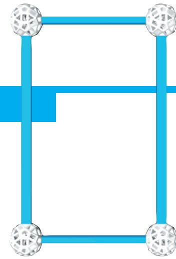
*Learn these **basic shapes** to help you build better models!*

*Flat, closed shapes are called **polygons**. They lie in a 2-D space (a plane). Why do they make boring bubbles?*

*A **regular** polygon has equal sides and equal angles.*

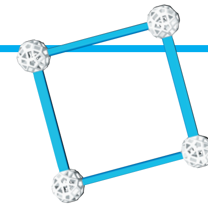
Red, yellow and blue struts lie in the blue plane.

*Models in the **Blue Plane** often show the number **2**! Every node has a **rectangular** hole facing up.*



2!

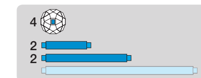
*A **Golden Rectangle** is like a 2-D number **2**. It has **2** sets of **2** different struts and **2x2** nodes. It also has **2**-fold symmetry.*



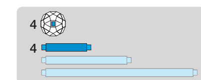
We know the rectangle and the square are in the blue plane, because every node has a rectangular hole facing up.

*A **Square** is a regular polygon, like a 2-D number **4 (2x2)**. It has **4** struts, **4** nodes, and **4**-fold symmetry.*

Golden Rectangle

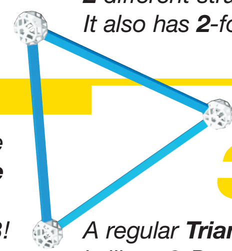


Square



Only blue struts lie in the yellow plane.

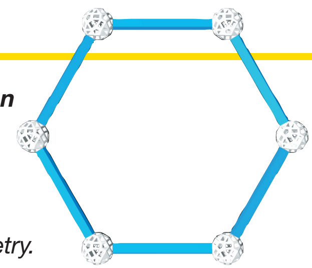
*Models in the **Yellow Plane** often show the number **3**! Every node has a **triangular** hole facing up.*



3!

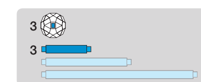
*A regular **Triangle** (**3**-sides) is like a 2-D number **3**. It has **3** equal struts and **3** balls. It also has **3**-fold symmetry.*

*A regular **Hexagon** (**6**-sides) is like a 2-D number **6 (3x2)**. It has **6** struts, **6** nodes, and **6**-fold symmetry.*

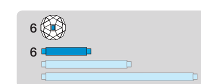


We know the triangle and the hexagon are in the yellow plane, because every node has a triangular hole facing up.

Triangle

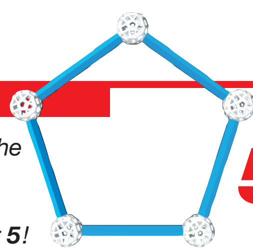


Hexagon



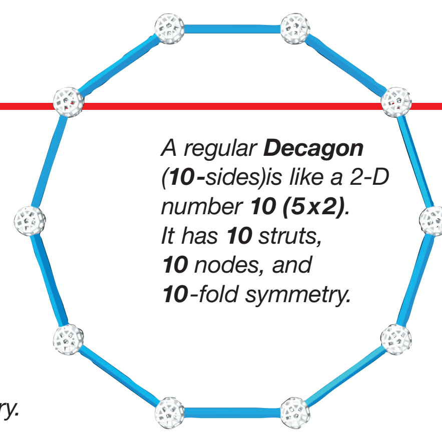
Only blue struts lie in the red plane.

*Models in the **Red Plane** often show the number **5**! Every node has a **pentagonal** hole facing up.*



5!

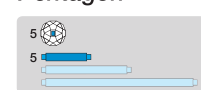
*A regular **Pentagon** (**5**-sides) is like a 2-D number **5**. It has **5** equal struts and **5** nodes. It also has **5**-fold symmetry.*



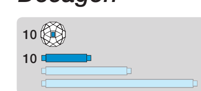
*A regular **Decagon** (**10**-sides)is like a 2-D number **10 (5x2)**. It has **10** struts, **10** nodes, and **10**-fold symmetry.*

We know the pentagon and the decagon are in the red plane, because every node has a pentagonal hole facing up.

Pentagon



Decagon

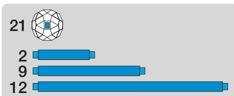


2, 3 & 5 in Nature

Many objects in Zometool have 2-, 3- and 5-fold symmetry. Can you find these relationships in nature?

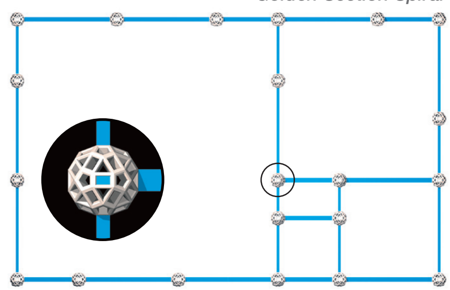
"To see a world in a grain of sand and heaven in a wild flower, hold infinity in the palm of your hand and eternity in an hour." —William Blake

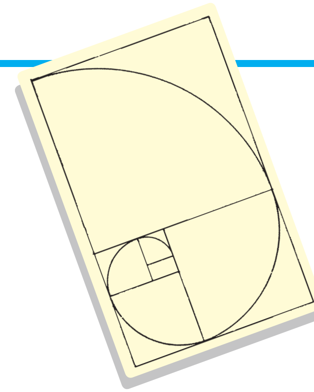
Golden Rectangle Spiral



***Fractal Symmetry** occurs when each part embodies the pattern of the whole.*

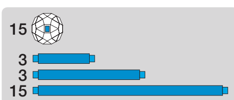
Golden Section Spiral



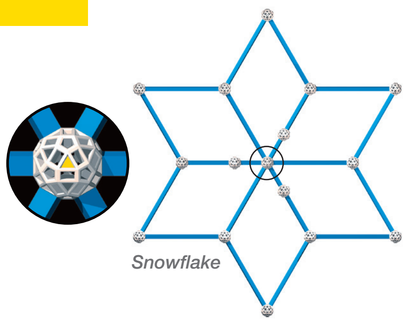


Just as the Golden Rectangle Spiral 'grows' in successive, related rectangles, so the nautilus shell grows in proportional repeating elements that build upon each other in stages. This process is common in many life forms.

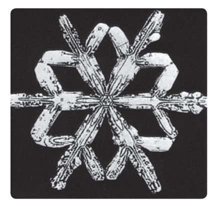
Snowflake



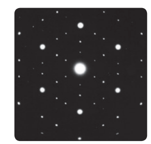
***Reflection Symmetry** occurs when applying a mirror plane to either of 2 halves recreates the whole.*



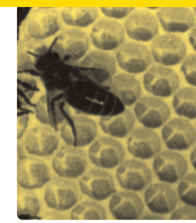
Snowflake



A snowflake has 6-fold rotational and reflection symmetry.

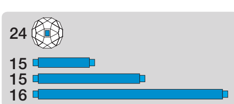


X-ray diffraction pattern of Al-Mn quasicrystal.



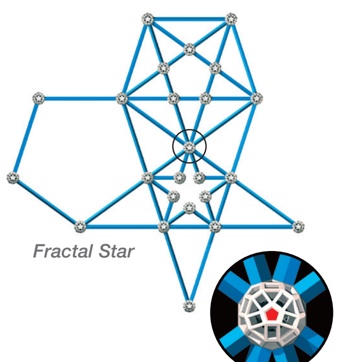
*A honeycomb is a **tiling** of hexagons. A tiling is a pattern that has **translational symmetry**, which occurs when the pattern repeats by shifting it a constant distance. (See Bee House model.)*

Fractal Star

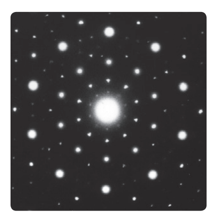


Elements of the fractal star exhibit reflection, rotational and fractal symmetries!

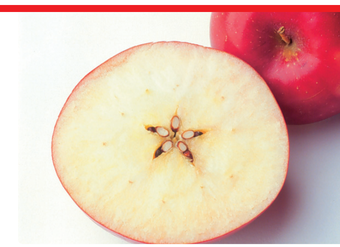
***Rotational Symmetry** occurs when an object rotated around its axis appears in the same position 2 or more times. The 5-pointed star has both rotational and reflection symmetries, but can also "grow" larger and smaller in fractal symmetry, like the Golden Rectangle.*



Fractal Star



*This **x-ray diffraction pattern** of a **quasicrystal** is full of the number 5. Can you see the pentagons and stars in it?*

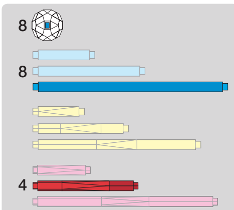


An apple cut on its equator has 5-fold rotational and reflection symmetry. So does a starfish!

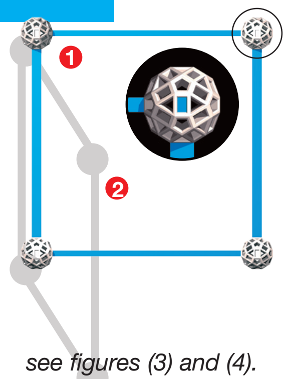
*Shadow is another way of saying **“projection”**.*

*Most shadows are flat (2-D) images of 3-D objects. With Zometool, you can build **3-D “shadows” of 4-D objects**. It’s easy!*

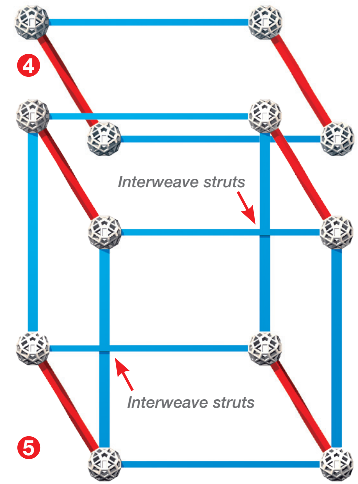
Impossible Cube



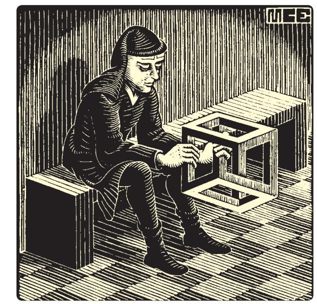
*If you hold up a square (1) and cast a shadow onto the floor, you can create a squashed shadow (2). You can **build** such shadows in Zometool:*



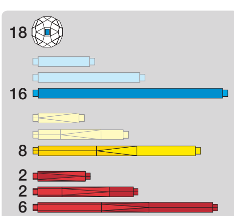
see figures (3) and (4). Together with the original square, these forms are “shadows” of the faces of a 3-D cube.

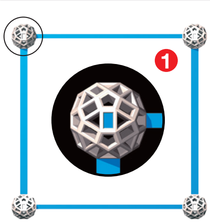


Combine the squashed squares (3) and (4) to build a 2-D shadow of the 3-D cube. By interweaving two sets of blue struts (5), you get an “Impossible Cuboid”.

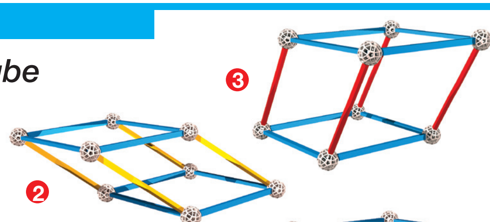


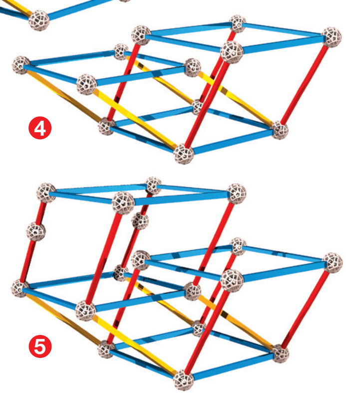
Parallel Hypercube Shadow



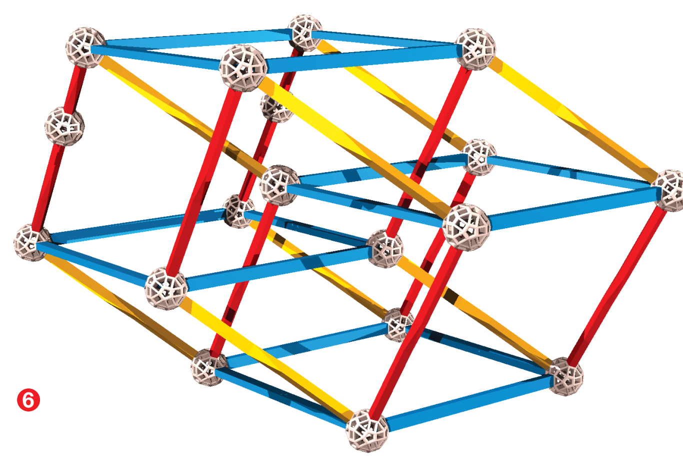


*Just as you built a shadow of a cube out of **squashed squares**, you can build a parallel shadow of a 4-D cube (6) out of squashed cubes (2) and (3)!*



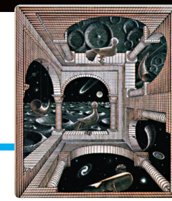


***Follow these steps** to discover how the squashed cubes fit together to make a parallel **3-D shadow** of a 4-D cube! How many cubes make a 4-D cube? Can you count them all in this shadow?*



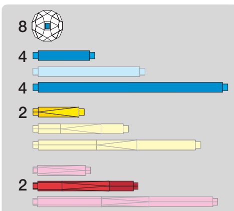
Shadows from the 4th Dimension

Follow these steps to discover how you can cast ***perspective "shadows"*** *of a 2-, 3- and 4-D cubes!*

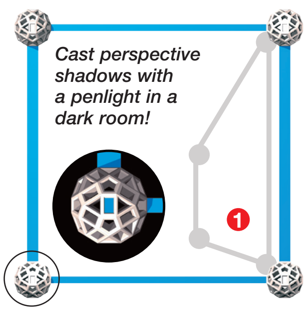


A perspective cube structure from "Another World" by M.C. Escher.

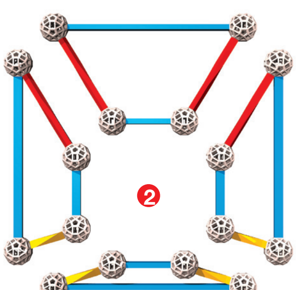
2-D Perspective Cube Shadow



Cast a perspective shadow of a square that looks like (1).



You can build shadows, or ***perspective squares*** *like these:*

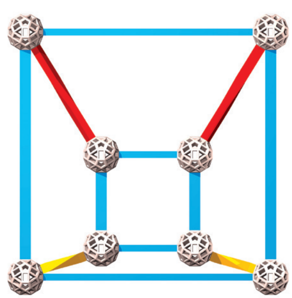


Using a penlight in a dark room, you can cast a perspective shadow of a regular cube that looks like (3).

3

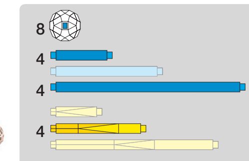
How many squares make a cube? Can you count them all in this shadow?

Now combine the four perspective squares that you already built (2) to form a ***perspective cube shadow*** *(4).*



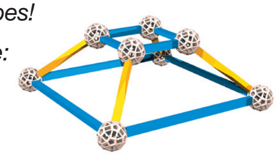
4

3-D Perspective Cube Shadow



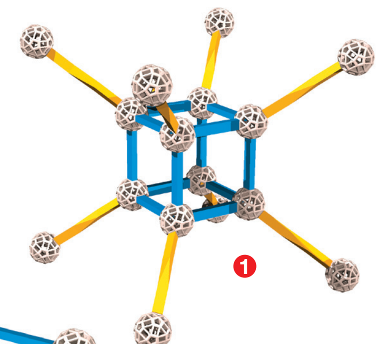
Just as you built a shadow of a cube out of perspective squares, you can build a ***perspective shadow of a 4-D cube*** *out of perspective cubes!*

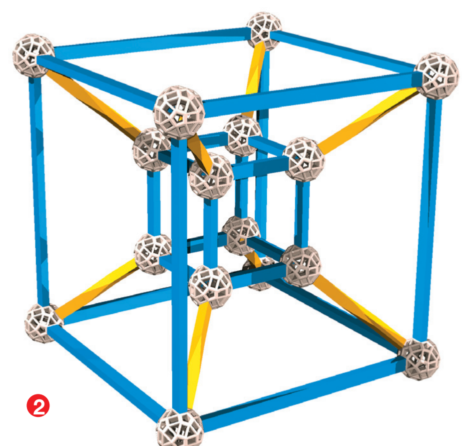
First build this one: it's a perspective ***3-D shadow*** *of a regular cube!*



Now build a ***4-D perspective cube shadow*** *by combining 3-D perspective cube shadows.*

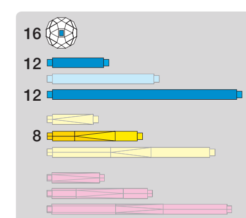
A 4-D cube is called a ***hypercube.*** *A hypercube can cast many different 3-D shadows. Compare this model (2) with figure (5) on page 15. How many cubes make a hypercube? Can you count them all in this shadow?*





Compare this perspective 4D cube shadow with the bubbles on the bottom of page 5!

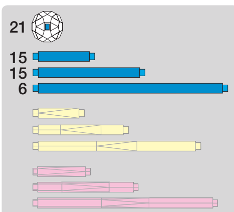
4-D Perspective Cube Shadow

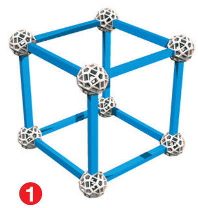


3-Dimensional Shapes

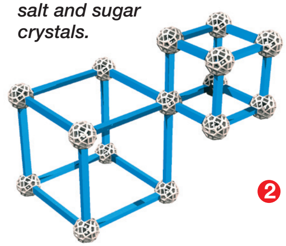
*3-Dimensional shapes are called **polyhedra** (many faces).*

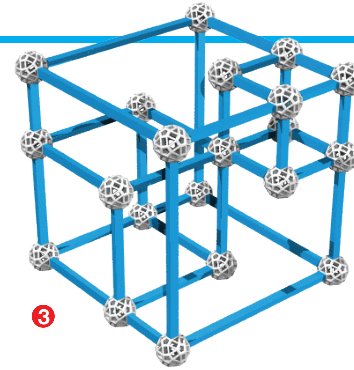
Fractal Cube



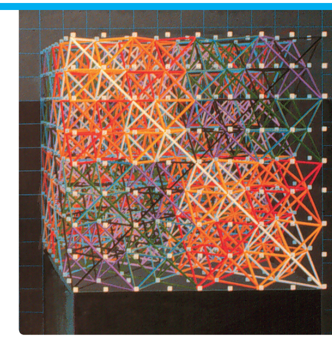


The cube often appears in natural forms, such as salt and sugar crystals.

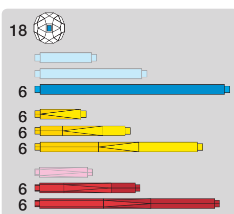




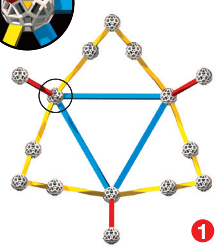
*Non-living crystals often take the form of a cube. This detail from the painting **"Octaval Complex"** by Clark Richert gives the periodic table of elements a pure geometric structure.*

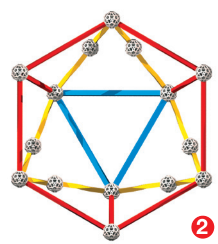


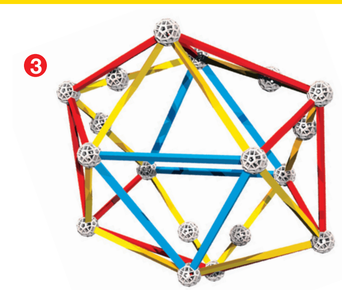
Squashed Virus



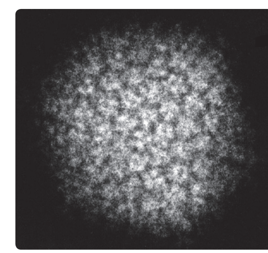
***Note:** Figures (1) and (2) are viewed from above; they are **not** in a plane. Refer to final model (3).*





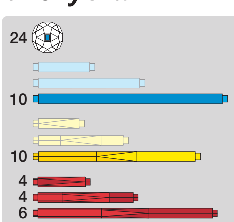


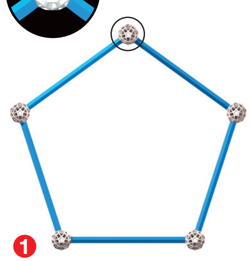
Squashed Virus (Icosahedron "squashed" along the 3-fold axis of symmetry).

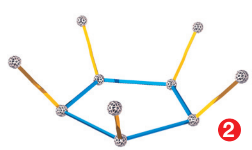


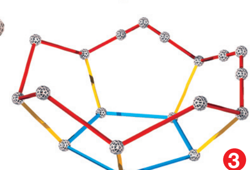
*Viruses often are related to the **icosahedron** (20-faces).*

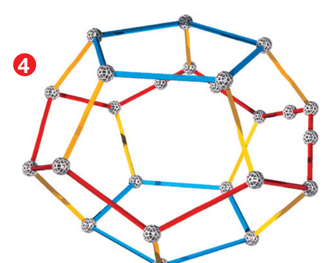
5-Crystal



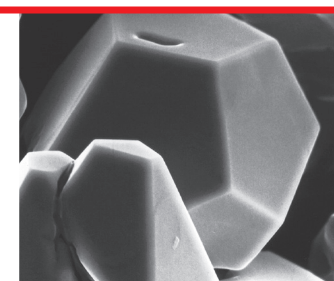








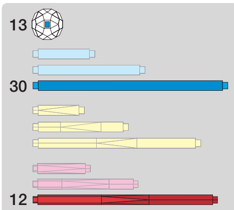
*5-Crystal is a squashed **dodecahedron** (12-faces).*

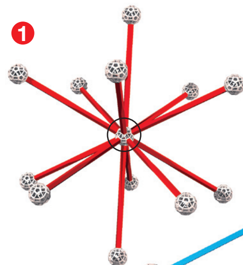


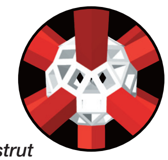
Scanning electron micrograph of quasicrystalline Al-Cu-Ru.

More 3-D Shapes

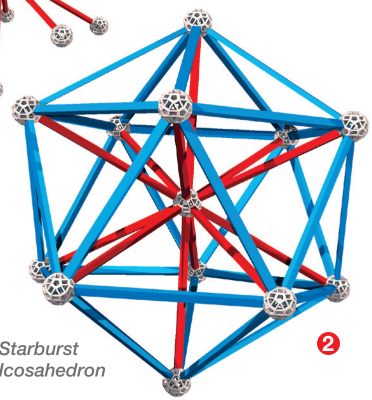
Starburst Icosahedron



1


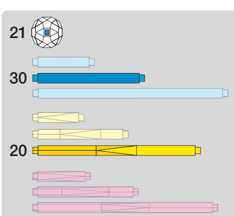


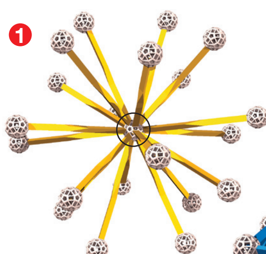
Note:
The central node has a long (#2) red strut in every pentagonal hole.



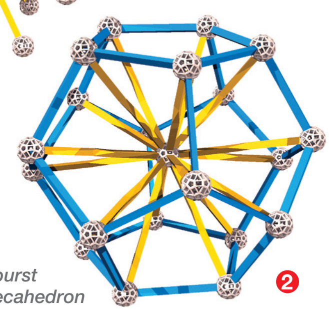
2
Starburst Icosahedron

Starburst Dodecahedron



1


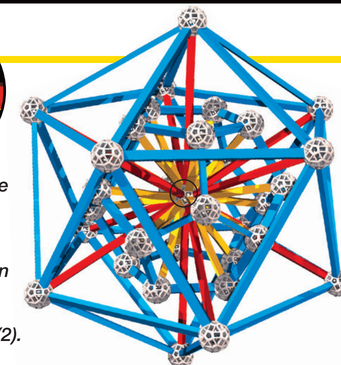
Note:
The central node has a long (#2) yellow strut in every triangular hole.

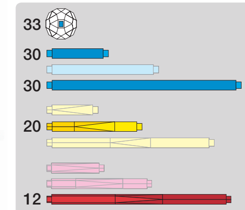


2
Starburst Dodecahedron

Double Starburst

Build a double starburst! It's a small starburst dodecahedron inside the starburst icosahedron (2).

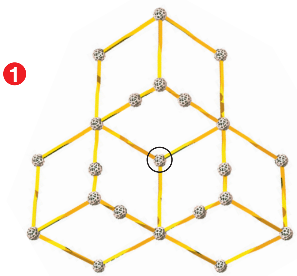


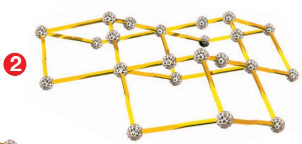


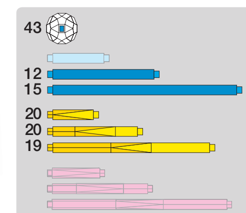
Note: *The central node has a medium (#1) yellow strut in every triangular hole and a long (#2) red strut in every pentagonal hole.*

Bee House

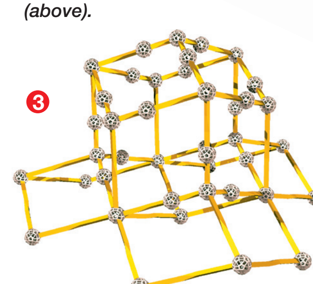
Note: *(1) shows the Bee House base from top view. Figure (2) shows the same base from a side view. Begin with the center node (above).*

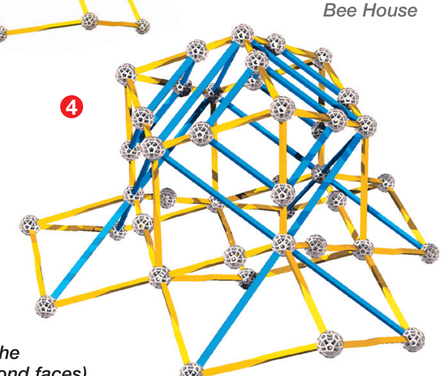
1


2




Bee House

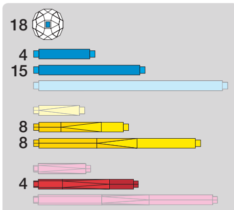
3


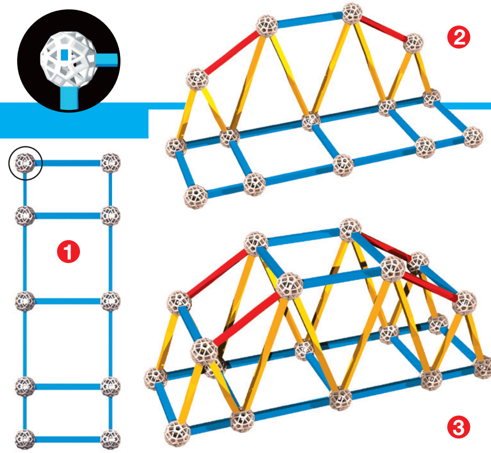
4


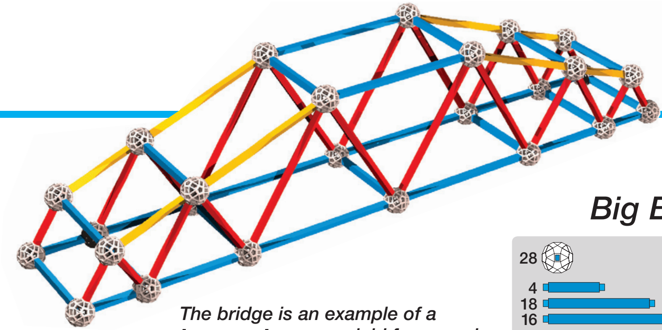
Bee House, like a real honeycomb, is based on a 3-D hexagon called the ***rhombic dodecahedron*** *(12 diamond faces).*

Structures!

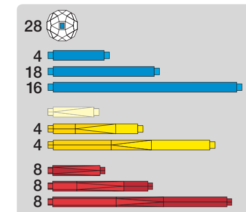
Little Bridge





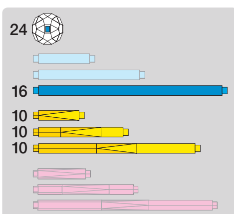


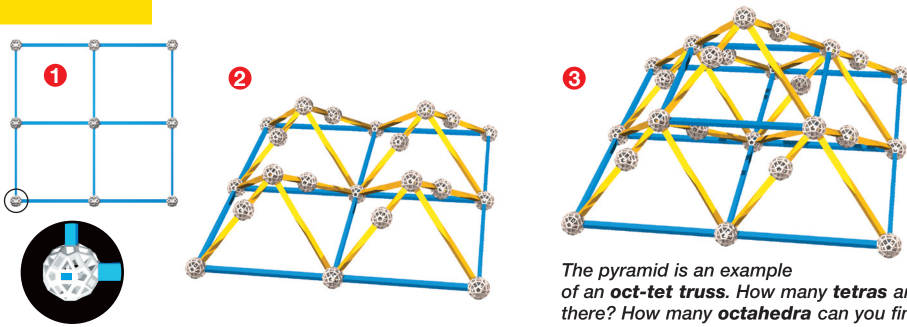
Big Bridge



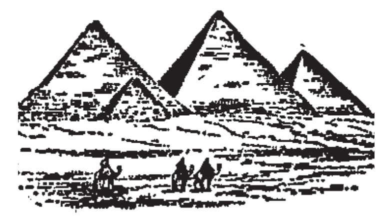
*The bridge is an example of a **truss system**, or a rigid framework that often shows a regular pattern. Can you invent your own bridge? What other architectural forms can you build using trusses? What is the tallest tower you can build?*

Pyramid



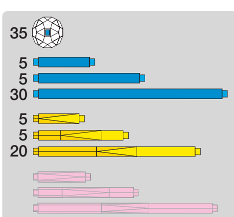


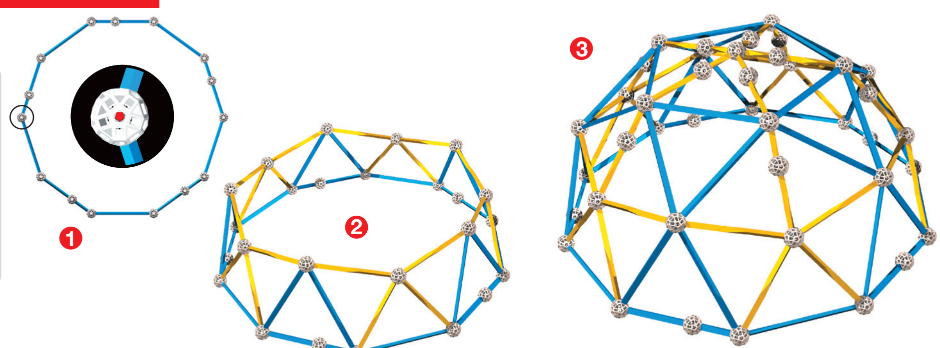
*The pyramid is an example of an **oct-tet truss**. How many **tetras** are there? How many **octahedra** can you find?*



Pyramids of Giza, Egypt

Dome





***La Géode** nears completion. This Buckminster Fulleresque dome, built in Paris in 1985, uses 1,670 steel triangles.*

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Acknowledgements

Manual Concept Development, Copywriting, Editing and Compilation **by Paul Hildebrandt, CVO, Zometool;**
Zometool Theory and Bibliography **by Marc Pelletier, Co-Founder, Zometool;**
Manual Design and Copywriting **by Dale Hess, Spark Studios;**
Zometool Logo, Identity and Branding **by Carlos Neumann, Chairman, Zometool**

Our Most Heartfelt Thanks to

Will Ackel: ***Computer Illustrations and Custom Ray Tracing Software*****;**
Scott Vorthmann, PhD: vZome software;
Steve Baer and Chris Kling: ***The Tetra Challenge Puzzle*****;**
W.A. Bentley and W.J. Humphreys: ***Snow Crystals,*** **Dover Publications, Inc., New York, 1962.*;***
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Magdolna and István Hargittai: ***Symmetry Text and Graphics*****;**
H.U. Nissen: ***Quasicrystal Electron Micrography;***
Clark Richert, artist: ***Detail from Octaval Complex;***
Geoffrey Wheeler Photography: ***Bubble Models and Life Forms Photographs ©1995*****;**
Robley C. Williams and Harold W. Fisher: ***Viruses Electron Micrography*****;**
Martin Wright: ***Scanning and 3-D Typography;***
Anni Wildung: ***layout and knowing everything;***
Mike Stranahan: ***Chief Zometool Angel***

31-zone system discovered by Stephen C. Baer, Zomeworks Corp., Albuquerque, NM USA.
GreenLines discovered by Clark Richert, Denver, CO USA.
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